

Spin-Orbit Correlations

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We summarize the intuitive connection between deformations of parton distributions in impact parameter space and single-spin asymmetries. Lattice results for the x^2 -moment of the twist-3 polarized parton distribution $g_2(x)$ are used to estimate the average transverse force acting on the active quark in SIDIS in the instant after being struck by the virtual photon.

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1. Impact Parameter Dependent PDFs and SSAs

The Fourier transform of the GPD $H_q(x, 0, t)$ yields the distribution $q(x, \mathbf{b}_\perp)$ of unpolarized quarks, for an unpolarized target, in impact parameter space [1]

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp}, \quad (1)$$

with $\Delta_\perp = \mathbf{p}'_\perp - \mathbf{p}_\perp$. For a transversely polarized target (e.g. polarized in the $+\hat{x}$ -direction) the impact parameter dependent PDF $q_{+\hat{x}}(x, \mathbf{b}_\perp)$ is no longer axially symmetric and the transverse deformation is described by the gradient of the Fourier transform of the GPD $E_q(x, 0, t)$ [2]

$$q_{+\hat{x}}(x, \mathbf{b}_\perp) = q(x, \mathbf{b}_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \quad (2)$$

$E_q(x, 0, t)$ and hence the details of this deformation are not very well known, but its x -integral, the Pauli form factor F_2 , is. This allows to relate the average transverse deformation resulting from Eq. (2) to the contribution from the corresponding quark flavor to the anomalous magnetic moment. This observation is important in understanding the sign of the Sivers function.

In a target that is polarized transversely (e.g. vertically), the quarks in the target nucleon can exhibit a (left/right) asymmetry of the distribution

$f_{q/p^\dagger}(x_B, \mathbf{k}_T)$ in their transverse momentum \mathbf{k}_T [3,4]

$$f_{q/p^\dagger}(x_B, \mathbf{k}_T) = f_1^q(x_B, k_T^2) - f_{1T}^{\perp q}(x_B, k_T^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_T) \cdot \mathbf{S}}{M}, \quad (3)$$

where \mathbf{S} is the spin of the target nucleon and $\hat{\mathbf{P}}$ is a unit vector opposite to the direction of the virtual photon momentum. The fact that such a term may be present in (3) is known as the Sivers effect and the function $f_{1T}^{\perp q}(x_B, k_T^2)$ is known as the Sivers function. The latter vanishes in a naive parton picture since $(\hat{\mathbf{P}} \times \mathbf{k}_T) \cdot \mathbf{S}$ is odd under naive time reversal (a property known as naive-T-odd), where one merely reverses the direction of all momenta and spins without interchanging the initial and final states. The momentum fraction x , which is equal to x_B in DIS experiments, represents the longitudinal momentum of the quark *before* it absorbs the virtual photon, as it is determined solely from the kinematic properties of the virtual photon and the target nucleon. In contradistinction, the transverse momentum \mathbf{k}_T is defined in terms of the kinematics of the final state and hence it represents the asymptotic transverse momentum of the active quark *after* it has left the target and before it fragments into hadrons. Thus the Sivers function for semi-inclusive DIS includes the final state interaction between struck quark and target remnant, and time reversal invariance no longer requires that it vanishes. Indeed, as time reversal not only reverses the

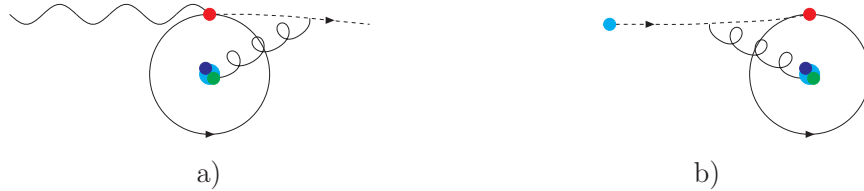


Fig. 1. In SIDIS (a) the ejected (red) quark is attracted by the (anti-red) spectators. In contradistinction, in DY (b), before annihilating with the (red) active quark, the approaching (anti-red) antiquark is repelled by the (anti-red) spectators.

signs of all spins and momenta, but also transforms final state interactions (FSI) into initial state interactions (ISI), it has been shown that the Sivers function relevant for SIDIS and that relevant for Drell-Yan (DY) processes must have opposite signs [5],

$$f_{1T}^{\perp}(x_B, k_T^2)_{SIDIS} = -f_{1T}^{\perp}(x_B, k_T^2)_{DY}, \quad (4)$$

where the asymmetry in DY arises from the ISI between the incoming antiquark and the target. The experimental verification of this relation

would provide a test of the current understanding of the Sivers effect within QCD. It is instructive to elucidate its physical origin in the context of a perturbative picture: for instance, when the virtual photon in a DIS process hits a red quark, the spectators must be collectively anti-red in order to form a color-neutral bound state, and thus attract the struck quark (Fig. 1). In DY, when an anti-red antiquark annihilates with a target quark, the target quark must be red in order to merge into a photon, which carries no color. Since the proton was colorless before the scattering, the spectators must be anti-red and thus repel the approaching antiquark.

The significant distortion of parton distributions in impact parameter space (2) provides a natural mechanism for a Sivers effect. In semi-inclusive DIS, when the virtual photon strikes a u quark in a \perp polarized proton, the u quark distribution is enhanced on the left side of the target (for a proton with spin pointing up when viewed from the virtual photon perspective). Although in general the final state interaction (FSI) is very complicated, we expect it to be on average attractive thus translating a position space distortion to the left into a momentum space asymmetry to the right and vice versa (Fig. 2) [6]. Since this picture is very intuitive, a few words of

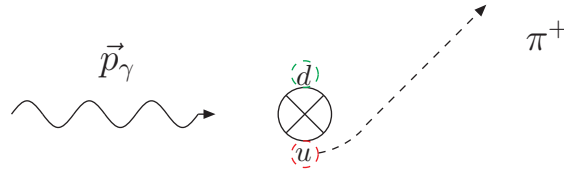


Fig. 2. The transverse distortion of the parton cloud for a proton that is polarized into the plane, in combination with attractive FSI, gives rise to a Sivers effect for u (d) quarks with a \perp momentum that is on the average up (down).

caution are in order. First of all, such a reasoning is strictly valid only in mean field models for the FSI as well as in simple spectator models [7]. Furthermore, even in such mean field models there is no one-to-one correspondence between quark distributions in impact parameter space and unintegrated parton densities (e.g. Sivers function). While both are connected by a Wigner distribution [8], they are not Fourier transforms of each other. Nevertheless, since the primordial momentum distribution of the quarks (without FSI) must be symmetric we find a qualitative connection between the primordial position space asymmetry and the momentum space asymmetry (with FSI). Another issue concerns the x -dependence of the Sivers function. The x -dependence of the position space asymmetry is

described by the GPD $E(x, 0, -\Delta_\perp^2)$. Therefore, within the above mechanism, the x dependence of the Siverts function should be related to the x dependence of $E(x, 0, -\Delta_\perp^2)$. However, the x dependence of E is not known yet and we only know the Pauli form factor $F_2 = \int dx E$. Nevertheless, if one makes the additional assumption that E does not fluctuate as a function of x then the contribution from each quark flavor q to the anomalous magnetic moment κ determines the sign of $E^q(x, 0, 0)$ and hence of the Siverts function. Making these assumptions, as well as the very plausible assumption that the FSI is on average attractive, one finds that $f_{1T}^{\perp u} < 0$, while $f_{1T}^{\perp d} > 0$. Both signs have been confirmed by a flavor analysis based on pions produced in a SIDIS experiment by the HERMES collaboration [9] and are consistent with a vanishing isoscalar Siverts function [10].

2. The Force on a Quark in SIDIS

The chirally even spin-dependent twist-3 parton distribution $g_2(x) = g_T(x) - g_1(x)$ is defined as

$$\begin{aligned} & \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma^\mu \gamma_5 \psi(\lambda n) | Q^2 | PS \rangle \\ &= 2 [g_1(x, Q^2) p^\mu (S \cdot n) + g_T(x, Q^2) S_\perp^\mu + M^2 g_3(x, Q^2) n^\mu (S \cdot n)] . \end{aligned}$$

neglecting m_q : $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$. $\bar{g}_2(x)$ involves quark-gluon correlations, e.g. [11,12]

$$\int dx x^2 \bar{g}_2(x) = \frac{d_2}{6} \quad (5)$$

with

$$g \langle P, S | \bar{q}(0) G^{+y}(0) \gamma^+ q(0) | P, S \rangle = M P^+ P^+ S^x d_2 \quad (6)$$

At low Q^2 , g_2 has the physical interpretation of a spin polarizability, which is why the matrix elements (note that $\sqrt{2} G^{+y} = B^x - E^y$)

$$\chi_E 2M^2 \vec{S} = \langle P, S | q^\dagger \vec{\alpha} \times g \vec{E} q | P, S \rangle \quad \chi_B 2M^2 \vec{S} = \langle P, S | q^\dagger g \vec{B} q | P, S \rangle \quad (7)$$

are sometimes called spin polarizabilities or color electric and magnetic polarizabilities [13]. In the following we will discuss that at high Q^2 a better interpretation for these matrix elements is that of a ‘force’.

As Qiu and Sterman have shown [14], the average transverse momentum of the ejected quark (here also averaged over the momentum fraction x

carried by the active quark) in a SIDIS experiment can be represented by the matrix element

$$\langle k_{\perp}^y \rangle = -\frac{1}{2P^+} \left\langle P, S \left| \bar{q}(0) \int_0^{\infty} dx^- G^{+y}(x^+ = 0, x^-) \gamma^+ q(0) \right| P, S \right\rangle \quad (8)$$

which has a simple physical interpretation: the average transverse momentum is obtained by integrating the transverse component of the color Lorentz force along the trajectory of the active quark — which is an almost light-like trajectory along the $-\hat{z}$ direction, with $z = -t$: The \hat{y} -component of the Lorentz force acting on a particle moving, with (nearly) the speed of light $\vec{v} = (0, 0, -1)$, along the $-\hat{z}$ direction reads

$$g\sqrt{2}G^{y+} = g(E^y + B^x) = g[\vec{E} + \vec{v} \times \vec{B}]^y. \quad (9)$$

We now rewrite Eq. (8) as an integral over time

$$\langle k_{\perp}^y \rangle = -\frac{\sqrt{2}}{2P^+} \langle P, S | \bar{q}(0) \int_0^{\infty} dt G^{+y}(t, z = -t) \gamma^+ q(0) | P, S \rangle \quad (10)$$

in which the physical interpretation of $-\frac{\sqrt{2}}{2P^+} \langle P, S | \bar{q}(0) G^{+y}(t, z = -t) \gamma^+ q(0) | P, S \rangle$ as being the averaged force acting on the struck quark at time t after being struck by the virtual photon becomes more apparent.

In particular,

$$\begin{aligned} F^y(0) &\equiv -\frac{\sqrt{2}}{2P^+} \langle P, S | \bar{q}(0) G^{+y}(0) \gamma^+ q(0) | P, S \rangle \\ &= -\frac{1}{\sqrt{2}} M P^+ S^x d_2 = -\frac{M^2}{2} d_2, \end{aligned} \quad (11)$$

where the last equality holds only in the rest frame ($p^+ = \frac{1}{\sqrt{2}}M$) and for $S^x = 1$, can be interpreted as the averaged transverse force acting on the active quark in the instant right after it has been struck by the virtual photon.

Lattice calculations of the twist-3 matrix element yield [15]

$$d_2^{(u)} = 0.010 \pm 0.012 \quad d_2^{(d)} = -0.0056 \pm 0.0050 \quad (12)$$

renormalized at a scale of $Q^2 = 5 \text{ GeV}^2$ for the smallest lattice spacing in Ref. [15]. Here the identity $M^2 \approx 5 \text{ GeV}^2/\text{fm}$ is useful to better visualize the magnitude of the force.

$$F_{(u)} = -25 \pm 30 \text{ MeV/fm} \quad F_{(d)} = 14 \pm 13 \text{ MeV/fm}. \quad (13)$$

In the chromodynamic lensing picture, one would have expected that $F_{(u)}$ and $F_{(d)}$ are of about the same magnitude and with opposite sign. The same

holds in the large N_C limit. A vanishing Sivers effect for an isoscalar target would be more consistent with equal and opposite average forces. However, since the error bars for d_2 include only statistical errors, the lattice result may not be inconsistent with $d_2^{(d)} \sim -d_2^{(u)}$.

The average transverse momentum from the Sivers effect is obtained by integrating the transverse force to infinity (along a light-like trajectory) $\langle k^y \rangle = \int_0^\infty dt F^y(t)$. This motivates us to define an ‘effective range’

$$R_{eff} \equiv \frac{\langle k^y \rangle}{F^y(0)}. \quad (14)$$

Note that R_{eff} depends on how rapidly the correlations fall off along a light-like direction and it may thus be larger than the (spacelike) radius of a hadron. Of course, unless the functional form of the integrand is known, R_{eff} cannot really tell us about the range of the FSI, but if the integrand does not oscillate

Fits of the Sivers function to SIDIS data yield 17 one finds about $|\langle k^y \rangle| \sim 100$ MeV [17]. Together with the (average) value for $|d_2|$ from the lattice this translates into an effective range R_{eff} of several fm. It would be interesting to compare R_{eff} for different quark flavors and as a function of Q^2 , but this requires more precise values for d_2 as well as the Sivers function.

Note that a complementary approach to the effective range was chosen in Ref. 18, where the twist-3 matrix element appearing in Eq. (11) was, due to the lack of lattice QCD results, estimated using QCD sum rule techniques. Moreover, the ‘range’ was taken as a model *input* parameter to estimate the magnitude of the Sivers function.

A measurement of the twist-4 contribution f_2 to polarized DIS allows determination of the expectation value of different Lorentz/Dirac components of the quark-gluon correlator appearing in (6)

$$f_2 M^2 S^\mu = \frac{1}{2} \langle p, S | \bar{q} g \tilde{G}^{\mu\nu} \gamma_\nu q | p, S \rangle, \quad (15)$$

In combination with (6) this allows a decomposition of the force into electric and magnetic components using

$$F_E^y(0) = -\frac{M^2}{8} \chi_E \quad F_B^y(0) = -\frac{M^2}{4} \chi_B \quad (16)$$

for a target nucleon polarized in the $+\hat{x}$ direction, where [13,16]

$$\chi_E = \frac{2}{3} (2d_2 + f_2) \quad \chi_M = \frac{1}{3} (4d_2 - f_2). \quad (17)$$

A relation similar to (11) can be derived for the x^2 moment of the twist-3 scalar PDF $e(x)$. For its interaction dependent twist-3 part $\bar{e}(x)$ one finds

for an unpolarized target [19]

$$4MP^+P^+e_2 = g \langle p | \bar{q} \sigma^{+i} G^{+i} q | P \rangle, \quad (18)$$

where $e_2 \equiv \int_0^1 dx x^2 \bar{e}(x)$. The matrix element on the r.h.s. of Eq. (18) can be related to the average transverse force acting on a transversely polarized quark in an unpolarized target right after being struck by the virtual photon. Indeed, for the average transverse momentum in the $+\hat{y}$ direction, for a quark polarized in the $+\hat{x}$ direction, one finds

$$\langle k^y \rangle = \frac{1}{4P^+} \int_0^\infty dx^- g \langle p | \bar{q}(0) \sigma^{+y} G^{+y}(x^-) q(0) | p \rangle. \quad (19)$$

A comparison with Eq. (18) shows that the average transverse force at $t = 0$ (right after being struck) on a quark polarized in the $+\hat{x}$ direction reads

$$F^y(0) = \frac{1}{2\sqrt{2}p^+} g \langle p | \bar{q} \sigma^{+y} G^{+y} q | p \rangle = \frac{1}{\sqrt{2}} MP^+ S^x e_2 = \frac{M^2}{2} e_2, \quad (20)$$

where the last identify holds only in the rest frame of the target nucleon and for $S^x = 1$.

The impact parameter distribution for quarks polarized in the $+\hat{x}$ direction was found to be shifted in the $+\hat{y}$ direction [20–22]. Applying the chromodynamic lensing model implies a force in the negative $-\hat{y}$ direction for these quarks and one thus expects $e_2 < 0$ for both u and d quarks. Magnitude: since $\kappa_\perp > \kappa$, expect odd force larger than even force and thus $|e_2| > |d_2|$.

It would be interesting to study not only whether the effective range is flavor dependent, but also whether there is a difference between the chirally even and odd cases. It would also be very interesting to learn more about the time dependence of the FSI by calculating matrix elements of $\bar{q} \gamma^+ (\partial^+ G^{+\perp}) q$, or even higher derivatives, in lattice QCD. Knowledge of not only the value of the integrand at the origin, but also its slope and curvature at that point, would be very useful for estimating the integral in Eq. (8).

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